A NEW FORMULATION OF THE MMFC TO AVOID THE NUMERICAL INSTABILITY

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ABSTRACT: The paper presents an improved version of the Modified Maximum Force Criterion (MMFC) published by Hora [1]. Hora's model cannot be used if the yield locus of the sheet metal contains straight segments. In such regions, the left branch of the predicted FLC shows a drop. The new formulation of the maximum force criterion postulates that the evolution of the sheet metal towards necking depends on the "distance" between the current strain state and the plane strain. The numerical tests show that the model presented in this paper is not affected by the numerical instabilities specific to Hora's model.

KEYWORDS: Sheet Metals, Forming Limit Curves, Maximum Force Criterion

1 INTRODUCTION

The concept of the forming limit curve (FLC) has been introduced by the Keeler and Backofen [2] and Goodwin [3], respectively, with the aim providing a simple and easily usable description of the sheet metal formability. FLC is a curve relating pairs of principal limit strains, which can be obtained at the surface of the sheet metal during a forming process prior to the occurrence of some defects (necking, fracture, etc.). Due to the simplicity, the FLC concept has been rapidly assimilated by the industry.

During the last five decades, the researchers have focused their efforts on the development of theoretical models for the accurate calculation of FLC's. The first mathematical formulations of such models published by Hill [4] and Swift [5] are based on the localized and diffuse necking hypotheses, respectively. Later on, Marciniak and Kuczynsky [6], as well as Hutchinson and Neale [7] have developed strain localization models based on the assumption that the necking is caused by a pre-existing thickness defect. In 1975, Storen and Rice [8] proposed the so-called "vertex theory" to describe the localized necking under biaxial stretching conditions.

In 1994 Hora and Tong [1] developed the so-called Modified Maximum Force Criterion (MMFC) with the aim of improving the diffuse necking model previously proposed by Swift. Their approach is based on the experimentally confirmed fact that the onset of necking significantly depends on the current strain ratio. Recently, the standard MMFC model has been improved by Hora and Tong [9] and Comsa et al. [10].

Aretz [11] notices that the modified maximum force criterion contains a mathematical singularity which emerges if the yield locus contains linear segments. In such cases, the predicted FLC presents a sudden drop at the level of the left branch.

The FLC predictions are strongly influenced by the shape of the yield locus used in the theoretical model (see Barlat [12]). The first anisotropic yield criterion was developed by Hill in 1948 [13]. Since then, many other yield criteria have been developed with the aim of obtaining a better description of the plastic anisotropy. A comprehensive presentation of the anisotropic yield criteria can be found in the monograph published by Banabic et al. [14].

The aim of this paper is to improve the MMFC model in order to fix the singularity issue. In general, the instability occurs when non-quadratic yield criteria are used in connection with the maximum force criterion. The numerical tests presented in this paper refer to a strongly anisotropic aluminium alloy (AA2090-T3). The plasticity of this material is described using a non-quadratic yield criterion (BBC2005).

2 THEORETICAL ASSUMPTIONS

The sheet metal is considered to behave as an orthotropic membrane under the plane-stress conditions

$$\begin{aligned} &\sigma_{i3} = \sigma_{3i} = 0, \quad i = 1,2,3, \\ &\dot{\varepsilon}_{j3} = \dot{\varepsilon}_{3j} = 0, \quad j = 1,2, \end{aligned} \tag{1}$$

involving the stresses and strain-rates expressed in the orthotropy frame (1, 2 and 3 are the indices associated to the rolling, transverse, and normal

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directions, respectively). One also assumes that the external loads do not produce tangential stresses and strains:

$$\sigma_{12} = \sigma_{21} = 0, \quad \dot{\varepsilon}_{12} = \dot{\varepsilon}_{21} = 0 \tag{2}$$

The non-zero stresses and strain-rates thus become principal values of the corresponding tensors. In order to emphasize their significance, the following notations will be used: $\dot{\varepsilon}_i = \dot{\varepsilon}_{ii}\,(i = 1,2,3)$ – principal strain rates, and $\sigma_j = \sigma_{jj}\,(j = 1,2)$ – principal stresses.

The mechanical response of the sheet metal will be described by a rigid-plastic model. The main ingredient of the constitutive model is the yield criterion:

$$\bar{\sigma}(\sigma_1, \sigma_2) = Y(\bar{\varepsilon}) \tag{3}$$

where $\bar{\sigma} = \bar{\sigma}(\sigma_1, \sigma_2) \geq 0$ is the equivalent stress (homogeneous function of the first degree), $\bar{\varepsilon} \geq 0$ is the equivalent (plastic) strain, and $Y = Y(\bar{\varepsilon}) > 0$ is the yield parameter controlled by a strictly increasing hardening law. The principal strain-rates are defined by the flow rule

$$\dot{\varepsilon}_j = \dot{\bar{\varepsilon}} \frac{\partial \bar{\sigma}}{\partial \sigma_j}, \quad j = 1,2, \tag{4}$$

and the incompressibility constraint

$$\dot{\varepsilon}_1 + \dot{\varepsilon}_2 + \dot{\varepsilon}_3 = 0 \tag{5}$$

In order to preserve the simplicity of the model, one assumes that the local state of the sheet metal evolves along linear load paths subjected to the constraint

$$\alpha = \sigma_2 / \sigma_1 = \text{const.}, \quad \sigma_1 > 0, \quad \sigma_1 \geq \sigma_2 \tag{6}$$

For any load state having the property given by Equation (6), $\bar{\sigma}$ and its partial derivatives with respect to the non-zero principal stresses could be expressed as follows:

$$\bar{\sigma} = \sigma_1 f(\alpha), \quad \frac{\partial \bar{\sigma}}{\partial \sigma_1} = g(\alpha), \quad \frac{\partial \bar{\sigma}}{\partial \sigma_2} = h(\alpha) \tag{7}$$

Equation (7) results from the first-degree homogeneity of the equivalent stress. The functions f, g, and h are only related to the particular formulation of the equivalent stress adopted in the model.

Equation (7) allows rewriting the yield criterion and the flow rule as follows (see Equations (3) and (4)):

$$\sigma_1 = Y(\bar{\varepsilon}) / f(\alpha), \tag{8}$$

$$\dot{\varepsilon}_1 = \dot{\bar{\varepsilon}} g(\alpha), \quad \dot{\varepsilon}_2 = \dot{\bar{\varepsilon}} h(\alpha) \tag{9}$$

One may prove that, under the constraint given by Equation (6), the strain path is also linear. As a consequence, Equation (9) can be easily integrated with respect to the time variable:

$$\varepsilon_1 = \bar{\varepsilon} g(\alpha), \quad \varepsilon_2 = \bar{\varepsilon} h(\alpha) \tag{10}$$

2.1 MMFC: STANDARD FORMULATION

The strain localization model proposed by Hora and Tong [1] postulates that the necking is preceded by an evolution of the sheet metal towards the plane strain (Figure 1).

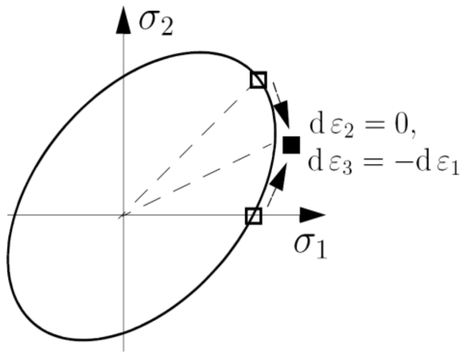

Fig. 1 *Evolution of the material towards the plane strain before the necking stage*

The failure condition found by Hora and Tong reads as follows:

$$\frac{\partial \sigma_1}{\partial \varepsilon_1} + \frac{\partial \sigma_1}{\partial \beta} \frac{\partial \beta}{\partial \varepsilon_1} = \sigma_1 \tag{11}$$

where $\beta \equiv \dfrac{\dot{\varepsilon}_2}{\dot{\varepsilon}_1} = const.$ is the ratio of the planar strain rates. According to Aretz [11], Equation (11) can be rewritten in the form

$$Y'(\bar{\varepsilon}) \cdot f(\alpha) \cdot g(\alpha) - Y(\bar{\varepsilon}) \cdot \frac{f'(\alpha) \cdot g(\alpha) \cdot \beta(\alpha)}{\beta'(\alpha) \cdot \bar{\varepsilon}} = \\ = f(\alpha) \cdot Y(\bar{\varepsilon}) \tag{12}$$

All the quantities having attached the prime symbol represent derivatives with respect to the parameter put within parentheses.

One may notice that if $\beta'(\alpha) = 0$, no solution for $\bar{\varepsilon}$ can be found. This is the mathematical singularity noticed by Aretz [11]. The problem will be fixed by the alternative model presented in the next section of the paper.

2.2 MMFC: NEW FORMULATION

Comsa et al. [10] proposed an alternate formulation of the maximum force criterion. According to their model, the necking occurs when the following equality is fulfilled:

$$\frac{\partial\sigma_1}{\partial\bar{\varepsilon}}\frac{\partial\bar{\varepsilon}}{\partial\varepsilon_1}+\frac{\partial\sigma_1}{\partial\gamma(\varepsilon_1,\alpha)}\frac{\partial\gamma(\varepsilon_1,\alpha)}{\partial\varepsilon_1}=\sigma_1 \qquad (13)$$

where $\gamma(\varepsilon_1,\alpha)$ is a measure of the "distance" separating the current state of the material from the plane-strain. The scalar quantity $\gamma(\varepsilon_1,\alpha)$ is defined by integrating the elementary arc-length of the normalized yield locus:

$$\mathrm{d}s=\sqrt{\left(\mathrm{d}\frac{\sigma_1}{Y}\right)^2+\left(\mathrm{d}\frac{\sigma_2}{Y}\right)^2} \qquad (14)$$

On the basis of the experimental evidence showing that the strain localization is preceded by the evolution of the material towards the plane-strain, the "distance" parameter $\gamma(\varepsilon_1,\alpha)$ is defined in the following manner:

$$\gamma(\varepsilon_1,\alpha)=\frac{1}{\varepsilon_1}\int_{\alpha_{FLC_0}}^{\alpha}\frac{\sqrt{g^2+h^2}}{f^2}\mathrm{d}\underline{\alpha} \qquad (15)$$

After some mathematical manipulations, the maximum force criterion defined by Equation (13) can be rewritten in the form

$$Y=\frac{1}{g}\left(\frac{\mathrm{d}Y}{\mathrm{d}\bar{\varepsilon}}+\frac{Y}{\bar{\varepsilon}}\frac{f\,h}{\sqrt{g^2+h^2}}\int_{\alpha_{FLC_0}}^{\alpha}\frac{\sqrt{g^2+h^2}}{f^2}\mathrm{d}\underline{\alpha}\right) \qquad (16)$$

This relationship allows the calculation of the equivalent strain associated to necking:

$$\bar{\varepsilon}=\frac{1}{g_\alpha}\left(n+\frac{f_\alpha h_\alpha}{\sqrt{g_\alpha^2+h_\alpha^2}}\int_{\alpha_{FLC_0}}^{\alpha}\frac{\sqrt{g_{\underline{\alpha}}^2+h_{\underline{\alpha}}^2}}{f_{\underline{\alpha}}^2}\mathrm{d}\underline{\alpha}\right) \qquad (17)$$

As soon as $\bar{\varepsilon}$ is known, the corresponding principal strains result from Equation (10). One may notice that the Equation (17) does not contain mathematical singularities. In fact, the "distance" parameter $\gamma(\varepsilon_1,\alpha)$ as defined by Equation (15) is a strictly increasing dependence on the principal strain ε_1. Under such circumstances, the second term in the left side of Equation (13) cannot vanish and the FLCs predicted by the new formulation of the maximum force criterion will not exhibit drops at the level of the left branch.

3 DISCUSSION REGARDING TO THE SINGULARITY PROBLEM

In order to demonstrate the fact that the new model fixes the singularity problem from the standard formulation of the MMFC model, the forming curve of an AA2090-T3 aluminium alloy has been calculated using both theoretical formulations. The same case has been analyzed by Aretz [11] in order to emphasize the singularity issue.
In this paper, the mechanical behaviour of the sheet metal is described by the non-quadratic BBC2005 yield criterion (see Banabic [14]). Due to the strong anisotropy of this aluminium alloy, the yield locus predicted by the BBC2005 constitutive models exhibits an extended linear segment in the first quadrant (see Figure 2).

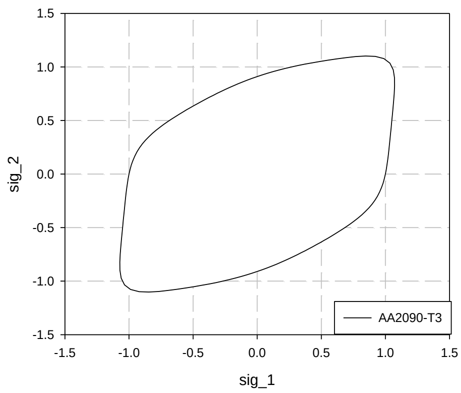

Fig. 2 *Yield locus predicted by BBC2005 yield criterion for the AA2090-T3 aluminium alloy*

The hardening law used in the FLC computation is given by Swift's power function [11]:

$$\mathrm{Y}\,(\bar{\varepsilon})=646\cdot(0.025+\bar{\varepsilon}\,)0.227\ \mathrm{N/mm}^2 \qquad (18)$$

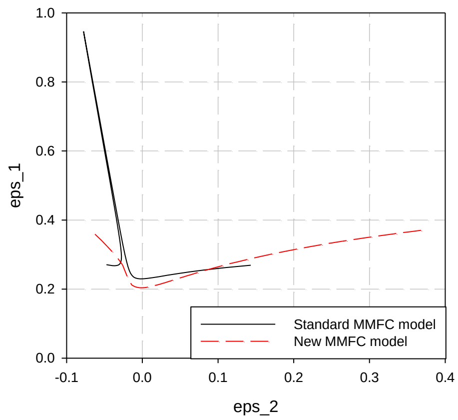

Fig. 3 *Comparison of the FLC's predicted using both maximum force models*

As noticeable in Figure 3, the curve predicted by the standard model contains a sudden drop at the level of the left branch. The predictions of the new maximum force criterion do not exhibit this defect. One may affirm that the mathematical singularity observed by Aretz [11] has been fixed in the new formulation of the model.

4 CONCLUSIONS

This paper analyzes the performances of a new formulation of the maximum force criterion. The numerical tests performed by the authors prove that the new model does not exhibit the numerical instabilities specific to the standard formulation proposed by Hora and Tong.

5 ACKNOWLEDGEMENT

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